AFET framework: ten open problems surveyed

No existing formalism treats entropy production Π as a running RG coupling, KZ scaling has never been applied to climate tipping, and multiple concrete counterexamples show Π alone cannot universally modulate criticality. These findings define both the theoretical frontier and the principal constraints for the AFET framework. Across all ten open problems, the literature reveals a consistent pattern: entropy production is a powerful diagnostic of non-equilibrium critical behavior, but it is neither sufficient nor monotonically related to proximity to phase transitions. The frenesy (time-symmetric dynamical activity) emerges as an indispensable complement to Π, Wikipedia and the Aron-Chamon non-analytic Landau theory offers the most promising formal pathway toward an entropy-modulated effective potential.

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1. No RG beta function for Π exists — a genuine open problem

The search for $\beta(\Pi) = d\Pi/d(\ln b)$ returned a definitive negative result. **No framework in the literature treats Π as a running coupling constant with its own RG beta function.** The existing work falls into three categories, none of which constitutes the target concept:

(a) RG flow of dissipation coefficients (standard). Canet, Chaté, Delamotte, and Wschebor (PRL 104, 150601, 2010; PRL 131, 247101, 2023) developed functional RG for the KPZ equation where running couplings are viscosity ν_κ, nonlinearity λ_κ, and noise strength D_κ — not Π. Sieberer, Huber, Altman, and Diehl (PRL 110, 195301, 2013; Rep. Prog. Phys. 79, 096001, 2016) showed in the Keldysh FRG for driven-dissipative BECs that arXiv the dissipation-to-coherence ratio $r = K_d/K_c$ flows under RG with a new critical exponent η_r, but again Π is not the running coupling.

(b) Π as a derived composite observable — closest to the target. Caballero and Cates (PRL 124, 240604, 2020) computed the scaling of EPR density σ near criticality in active φ^4 theories. Activity parameters are **RG-irrelevant** at the Ising critical point, yet EPR acquires a nontrivial critical exponent. DOI +2 Paoluzzi (PRE 105, 044139, 2022) found **$\theta_\sigma = \nu(4-d)$** for active Model A with colored noise. ResearchGate +2 In both cases, σ's scaling dimension is inferred from the scaling of its constituent operators — σ is not promoted to a running coupling in theory space.

(c) Entropy of the RG flow itself. Koenigstein et al. (PRD 106, 065013, 2022) showed the Wetterinck flow equation has intrinsic dissipative character. arXiv Cotler and Rezchikov (PRD 108, 025003, 2023) proved Polchinski's ERG equation is equivalent to optimal transport gradient flow of relative entropy. DOI Resolver These concern the mathematical properties of the RG transformation, not physical Π.

The conceptual innovation required would involve: identifying Π as an independent direction in theory space, including it in the truncation ansatz for the effective average action Γ_κ, and deriving a self-consistent flow equation where Π's running feeds back into other couplings. One could in principle construct $d\Pi/d(\ln b) = \Sigma_i (\partial\Pi/\partial g_i)\cdot\beta(g_i)$ from Nardini et al. (PRX 7, 021007, 2017), but this "induced flow" differs fundamentally from Π being an independent coupling. American Physical Society

Formal status: OPEN. AFET's proposed coupling of Π to threshold parameters would be genuinely novel.

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2. EPR at the directed percolation critical point is finite with a β-exponent cusp

Tomé and de Oliveira (J. Stat. Mech. 093202, 2024; arXiv:2405.06751) numerically computed the EPR for the 1d contact process (the paradigmatic DP-class model), resolving a technical challenge: the standard Schnakenberg formula diverges when reverse transition rates vanish at absorbing states. They proposed a modified expression (arXiv +2) $\Pi_{ij} = W_{ij} P_j \ln(P_j/P_i) - W_{ij}(P_j - P_i)$, which is positive-definite and finite.

The key finding: **EPR per site ψ is FINITE and nonzero at p_c,** (arXiv) with singular behavior $\psi(p) = \psi_c + A|p - p_c|^\beta + \ldots$, where **$\beta \approx 0.2765$** is the standard 1d DP order parameter exponent. The derivative $d\psi/dp$ diverges as $|p - p_c|^{\beta-1}$. (arXiv +2) In the absorbing phase ($p > p_c$, thermodynamic limit), $\psi \to 0$ trivially.

This corrects the naive expectation that $\Pi \sim \rho_{stat} \sim |p - p_c|^\beta$. The singular part tracks ρ_{stat}, but **Π has a nonzero regular background** at criticality — analogous to how specific heat can have a finite value plus a singular part governed by exponent α. There is **no independent EPR critical exponent**; it reduces entirely to the standard DP set $\{\beta, \nu_\perp, \nu_\parallel, z\}$. (Emergent Mind)

Complementary work by Noa, Harunari, de Oliveira, and Fiore (PRE 100, 012104, 2019) found EPR "portraits" at continuous and discontinuous nonequilibrium transitions in Z_2 systems: continuous transitions show a cusp in EPR; discontinuous ones show a jump. (APS Journals) (arXiv) Konishi and Iida (PRL 123, 090601, 2019) studied Rényi entropy (information-theoretic, not thermodynamic) at DP, finding universal relaxation behavior governed by the absorbing state. (APS Journals)

Quantity	Exponent	Status
Singular part of ψ near DP	β (DP order parameter)	Numerically verified (d=1)
dψ/dp divergence	β − 1	Numerically verified (d=1)
Independent EPR exponent	None — reduces to DP β	Verified
Field-theoretic (JdD) derivation	Not computed	**Open gap**
d > 1 or other absorbing-state classes	Not computed	**Open gap**

AFET implication: The coupling operator F must accommodate Π being finite at criticality with a singular correction, not vanishing or diverging.

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3. Non-Markovian coarse-graining can violate EP monotonicity

For Markovian systems, $\sigma_{CG} \leq \sigma_{full}$ is rigorously proven via the data processing inequality (Esposito, PRE 85, 041125, 2012). For non-Markovian systems, multiple pathways to violation exist:

Negative EP rates as a non-Markovianity witness. Strasberg and Esposito (PRE 99, 012120, 2019) proved that $d\Sigma/dt < 0$ implies non-Markovian dynamics for classical systems initialized in conditional steady states. (arXiv) The integrated EP $\Sigma(t)$ remains non-negative overall but can decrease transiently. For quantum non-Markovian maps, even integrated EP can go negative (Marcantoni et al., Sci. Rep. 7, 12447, 2017). (Nature)

Dramatic EP loss in GLE coarse-graining. Puglisi and Villamaina (EPL 88, 30004, 2009) showed that when a bipartite Markovian system is reduced to a generalized Langevin equation with linear forces, **the average EP vanishes entirely** — a far more severe loss than standard state-space coarse-graining. The harmonic GLE does not produce entropy regardless of work protocol (Martins et al., PRE 103, 022108, 2021); only nonlinearity recovers entropy-producing behavior.

Time-delayed systems extract entropy from baths. Loos and Klapp (Sci. Rep. 9, 2491, 2019) proved that overdamped systems with time-delayed feedback can achieve **negative medium entropy production** — delay forces literally refrigerate the bath. Nature Nature Discrete time delay produces divergent EP (Loos and Klapp, New J. Phys. 2020), related to the infinite-dimensional delay embedding.

Non-monotonic behavior across resolution levels. Bilotto, Caprini, and Vulpiani (PRE 104, 024140, 2021) showed that the overdamped approximation can produce either excess or deficit EP relative to underdamped truth, depending on friction strength. PubMed APS Journals

A fundamental open challenge flagged by Dieball and Godec (J. Chem. Phys. 162, 090901, 2025): **how to treat memory under time reversal** is generally unknown. Memory effects possess some oddness under time reversal but one cannot simply flip the sign. AIP Publishing

The Falasco-Esposito scaling law $\sigma(bL) = b^\alpha \sigma(L)$ has **no known non-Markovian extension**. For power-law memory kernels $K(t) \sim t^{-\gamma}$ (fractional Brownian motion, Mittag-Leffler), the scaling exponent α's dependence on γ is entirely open.

AFET implication: The hierarchical EP decomposition across levels requires careful treatment. Information flow terms must supplement EP at each level: $\sigma_{full} = \sigma_{CG} + \sigma_{hidden} + \dot{I}_{flow}$ (Loos and Klapp, 2020). For AFET's multi-scale structure, this means the coupling operator F must include information-theoretic corrections beyond Π alone.

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4. Frenesy has been measured in colloidal and mesoscopic systems but not in complex domains

The Maes decomposition splits path probability into time-antisymmetric (entropy production) and time-symmetric (frenesy/dynamical activity D) sectors. Kuleuven +3 Direct experimental measurements of D exist only in controlled mesoscopic settings:

Measured systems. Brandner, Maisi, Pekola, Garrahan, and Flindt (PRL 118, 180601, 2017) measured dynamical activity through counting statistics of Andreev tunneling events in a superconductor-normal metal device, extracting dynamical Lee-Yang zeros from high-order cumulants. American Physical Society Worktribe Blickle, Speck, Lutz, Seifert, and Bechinger (PRL 98, 210601, 2007) measured the frenetic contribution to the generalized Einstein relation in a driven colloidal system. For molecular motors, Ariga et al. measured "hidden dissipation" in kinesin — internal chemical transitions contributing to D but invisible as spatial steps. PubMed Central

Neural systems: proxy observables identified, no direct measurement. The comprehensive review by Nartallo-Kaluarachchi et al. (Physics Reports, 2025; arXiv:2504.12188) covers non-equilibrium brain dynamics but focuses exclusively on irreversibility, not frenesy. ScienceDirect Measurable proxies include total spike counts (traffic), ion channel switching rates (patch clamp), and brain-state switching rates from HMM fits to fMRI/EEG data. Maes (arXiv:2212.06211) explicitly proposed frenetic steering for pattern recognition in nonequilibrium neural network models: under strong metabolic driving, pattern storage uses time-symmetric transition rate modification, not free energy landscapes. arXiv

Biological networks: unmeasured as D but accessible. Single-molecule FRET tracks conformational switching (total transitions = traffic). ^{13}C metabolic flux analysis gives exchange fluxes (undirected traffic). Kinetic proofreading is precisely an example of increased frenesy enabling error correction — the discrimination ratio depends on the frenetic contribution.

Climate/geophysical systems: no measurement framework. Plausible proxies include weather pattern switching rates (blocking ↔ zonal flow), eddy kinetic energy (EKE, measurable from satellite altimetry), and total seismic moment release rate. Maes and collaborators (Physica A 552, 122179, 2020) discussed nonequilibrium approaches potentially relevant to geophysical systems.

The critical practical limitation: **D requires kinetic information (transition rates, trajectory data) while Π can be accessed calorimetrically.** ScienceDirect Coarse-graining hides D more than Π. Frontiers ScienceDirect For AFET, this means the D_d term in the coupling operator will be the hardest to measure in complex systems.

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5. Kibble-Zurek entropy scaling has never been applied to climate tipping

After exhaustive search, **zero publications** apply the Deffner-Zurek (PRE 96, 052125, 2017) scaling APS ⟨Σ_irrev⟩ ~ c^{2/(1+zν)} to any climate subsystem. No universality class assignments exist for any tipping element. No values of z or ν have been extracted from climate data.

The fundamental obstacle is that most climate tipping points are saddle-node bifurcations, not continuous phase transitions. Van Westen and Dijkstra (Earth Syst. Dynam. 16, 2063, 2025) confirm the AMOC follows a saddle-node bifurcation with square-root dependence on freshwater forcing. Copernicus Boulton, Lenton, and Boers (Nature Clim. Change 12, 271, 2022) detected resilience loss in the Amazon consistent with a fold bifurcation. Boers and Rypdal (PNAS 118, e2024192118, 2021) found critical slowing down in Greenland ice sheet data PubMed modeled via saddle-node normal form. Bochow et al. (Nature, 2023) showed substantial hysteresis — characteristic of first-order transitions. PubMed

For the generic saddle-node dx/dt = μ − x², the timescale diverges as τ ~ |μ − μ_c|^{−1/2}. **There is no divergent correlation length, no universality class, and no z or ν in the thermodynamic sense.** The KZ mechanism requires a continuous transition with well-defined critical exponents.

The closest climate analog is **rate-induced tipping** (R-tipping, Ashwin, Wieczorek, Vitolo, Cox, Phil. Trans. R. Soc. A 370, 1166, 2012), where the system fails to track a moving quasi-steady state when forcing rate exceeds a critical rate. Lohmann and Ditlevsen (PNAS 118, e2017989118, 2021) demonstrated rate-induced AMOC collapse. Copernicus R-tipping shares a conceptual parallel with KZ (both involve rate-dependence) but differs fundamentally: no critical point is traversed.

⚠️ **AFET constraint:** If the framework relies on KZ entropy scaling for climate applications, it faces a serious scope limitation. A modified theory for "entropy production scaling at finite-rate saddle-node passage" would be needed — likely yielding different functional forms involving the saddle-node ghost timescale rather than z and ν.

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6. Thermodynamic and information-theoretic entropy are formally linked but loosely so at neural criticality

Three distinct quantities must not be conflated: thermodynamic EP Π_thermo (J/K/s from ATP hydrolysis), information-theoretic EP Π_info (bits/s from broken detailed balance), and Shannon entropy rate h (bits/s of spike trains). When local detailed balance holds with a single thermal bath at temperature T, **Π_thermo = k_B T × Π_info** exactly — but this requires Markovian dynamics and complete observation of all relevant degrees of freedom.

Aguilera, Igarashi, and Shimazaki (Nature Comm. 14, 3685, 2023) provide the most directly relevant result. In the asymmetric SK model (exact solution at N → ∞), EP shows a **local peak at the critical order-disorder line** but reaches its **global maximum in quasi-deterministic disordered regimes** at low temperature with high coupling heterogeneity — far from criticality. Miguel Aguilera +3 This exactly solved result directly falsifies any simple monotonic Π ↔ criticality mapping.

Formal bounds linking Π and h exist but are extremely loose for biology:

- **Landauer bound**: Π ≥ k_BT ln(2) × R_erase. At body temperature, k_BT ln(2) ≈ 3 × 10^{-21} J/bit. Wikipedia Actual synaptic transmission costs **~10^5 k_BT/bit** (Laughlin et al., Nature Neuroscience 1998; Karbowski, Entropy 2024) — operating 10^4–10^7 × above the bound. nih
- **Still-Sivak theorem** (PRL 109, 120604, 2012): W_diss = k_BT × I_nostalgia, where I_nostalgia is nonpredictive information. This provides a formal, exact link between dissipation and the system's failure to be predictive.
- **Thermodynamic uncertainty relation** (Barato and Seifert, 2015): σ̇ ≥ 2⟨J⟩²/Var(J), bounding EP from below by current precision.

Sekizawa, Ito, and Oizumi (PRX 14, 041003, 2024) decomposed housekeeping EP into oscillatory mode contributions: arXiv **σ_hk = Σ_k ω_k² P_k**, applied to monkey ECoG data. arXiv ADS Lynn, Cornblath, Papadopoulos, Bertolero, and Bassett (PNAS 118, e2109889118, 2021) measured broken detailed balance from fMRI: brains break detailed balance more during tasks than at rest. PNAS arXiv

Simulation evidence suggests criticality optimizes the **energy cost per bit of communicated information** rather than minimizing or maximizing total dissipation. Critical avalanche dynamics produce sublinear metabolic scaling matching Kleiber-like laws (arXiv:2512.10834, 2024). Roberts et al. (Front. Syst. Neurosci. 8:166, 2014) proposed metabolic depletion during intense activity as a negative feedback loop maintaining neural criticality. PubMed

Bottom line: Formal links between Π_thermo and h exist via Landauer, Still-Sivak, and stochastic thermodynamics, but they are extremely loose for biological neural systems. **No direct calorimetric measurement of Π at σ = 1 vs. σ ≠ 1 exists.** The Aguilera et al. result that EP is globally maximized away from criticality is a significant AFET constraint. PubMed Central

7. Aron-Chamon non-analytic NESS potential and the pathway to entropy-modulated effective potentials

Aron and Chamon (SciPost Phys. 8, 074, 2020) demonstrated that breaking detailed balance in a mean-field spin system generates non-analytic terms in the NESS effective potential:

$V\_NESS(\varphi) = a_2\varphi^2 + a_4\varphi^4 + c\_\alpha|\varphi|^{2+\alpha}$ scipost

The exponent α is determined by the low-energy spectrum of the dissipative environment: **$\alpha = d_2/z_2 - d_1/z_1$** for a system coupled to two baths with dimensionality d_n and dynamical exponent z_n. SciPost The coefficient c_α **vanishes at equilibrium** and is a smooth function of bath temperatures. scipost For 0 < α < 2, the non-analytic term dominates over φ^4 near the transition, yielding a **new universality class with modified mean-field exponents**. SciPost

The follow-up paper by Aron, Kulkarni, and Chamon (PRR 2, 043390, 2020) studied an Ising magnet undergoing stochastic reheating, performing explicit RG analysis and identifying a **new nonequilibrium fixed point** where non-analytic operators are infrared relevant. Most recently, Ptaszyński and Esposito (PRE 111, 044142 and PRE 111, 034125, 2025) extended the framework to finite-size scaling and heat current fluctuations in the nonequilibrium Curie-Weiss model, arXiv arXiv directly building on the Aron-Chamon foundation. SciPost

The c_α ↔ Π link is implicit, not formal. Both c_α and Π vanish at equilibrium and are controlled by the same non-equilibrium parameters (temperature difference between baths), but no paper derives c_α = f(Π). This is the most significant gap for AFET: deriving this functional relationship would establish a formal pathway from Π to effective potential modification. Additional open extensions include: conserved order parameter (Model B), coupled fields, and active matter applications. SciPost

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8. Fisher information connects to sigmoid structure through information geometry, but no unified derivation exists

The Wasserstein-2 gradient flow framework (Jordan-Kinderlehrer-Otto, 1998) produces the Fokker-Planck equation as gradient flow of the free energy, with **EPR = Fisher information $I(\rho) = \int\rho|\nabla\ln(\rho/\rho\_eq)|^2\,dx$**. Ito (PRL 121, 030605, 2018; Sosuke110 PRR 3, 043093, 2021; ResearchGate Information Geometry 2023) developed the full "geometric thermodynamics" connecting Wasserstein distance, Fisher metric, and stochastic thermodynamics: ResearchGate entropy production rate equals half the Fisher metric, and total EP equals the W_2 action along the trajectory. ResearchGate

The connection to sigmoid functions is structural but not unified into a single theorem:

- **Fisher information of Bernoulli(σ(θ)) = σ'(θ) = σ(θ)(1−σ(θ))** — the sigmoid derivative IS the Fisher information for a binary system. Status: **derived** (elementary information geometry).
- **Natural gradient on sigmoid-parametrized manifolds** (Amari, 1998): Near a phase transition ($\sigma \approx 1/2$), the Fisher metric is maximal; far from transition ($\sigma \approx 0$ or 1), the natural gradient correction diverges. Status: **derived**.
- **Fisher information peaks at phase transitions** in bistable systems (Kim et al., Entropy 19, 268, 2017). Status: **formal** (computed numerically).

The chain of reasoning connecting EPR to sigmoid dynamics goes: Wasserstein gradient flow → Fokker-Planck → bistable dynamics → Kramers rates → logistic/sigmoid switching for coarse-grained occupation probability. Each link is established, but **no single derivation connects the W_2 gradient flow directly to sigmoid activation**. The structural argument that EPR's Fisher information geometry on a coarse-grained bistable manifold naturally produces sigmoid activation is plausible but unproven. This represents a clear opportunity for AFET to provide the missing formal bridge.

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9. Five classes of counterexamples constrain AFET significantly

The falsification search identified serious counterexamples across all three categories:

High EP without criticality. The Aguilera et al. (2023) exact solution of the asymmetric SK model is the most damaging single result: EP is **significantly larger** in quasi-deterministic disordered regimes than at the critical line. Nature +3 Landauer's blowtorch theorem (Phys. Rev. A 12, 636, 1975) proved that entropy production and its derivatives are inadequate to characterize steady states in bistable systems — kinetic (frenetic) factors determine occupations. Fully developed turbulence sustains enormous Π without a well-defined phase transition in the developed regime.

Transitions at zero or negligible EP. Quantum phase transitions at $T = 0$ (Sachdev, 2011) are genuine critical phenomena with diverging correlation lengths and no thermal dissipation. Topological phase transitions (BKT, Chern insulator) change global invariants without local order parameters or EP. The many-body localization transition occurs in isolated quantum systems with $\Pi = 0$ — and introducing dissipation (dephasing) actively destroys the MBL phase (Žnidarič, 2015).

Increasing dissipation suppresses criticality. The quantum Zeno effect — strong measurement freezing dynamics — destroys all critical correlations (Turkeshi et al., Quantum 5, 528, 2021). Markovian dissipation suppresses quantum criticality in the Lipkin-Meshkov-Glick model; only non-Markovian (structured) environments preserve it (arXiv:2504.11317, 2025). Strong loss destroys superfluid order non-analytically at finite critical time (arXiv:2506.05770, 2025).

Dissipation-induced transitions that partially support AFET. Dissipation can create topological transitions where the Z_2 invariant depends exclusively on dissipation operators (Deng and Yang, PRB 113, 024312, 2026). Dissipation stabilizes discrete time crystal order (Nakamura et al., PRR 4, 023025, 2022). APS Journals These show dissipation CAN enable transitions, but the mechanism is structure-specific.

Maes' frenesy as the missing ingredient. Maes (Physics Reports 850, 1, 2020) proved that entropy production alone is theoretically insufficient for characterizing nonequilibrium steady states. Both the time-antisymmetric (Π) and time-symmetric (D) sectors are required. ScienceDirect +2 This is not merely a practical limitation but a theorem-level result grounded in the Landauer blowtorch theorem.

Counterexample	Category	Severity for AFET
Aguilera et al. 2023 (EP max away from criticality)	A	★★★★★
Quantum phase transitions at T = 0	B	★★★★★
Quantum Zeno effect	C	★★★★★
Landauer blowtorch (Π insufficient for NESS)	A	★★★★
Markovian dissipation kills LMG criticality	C	★★★★
MBL transition (Π = 0; dissipation destroys it)	B+C	★★★★
Maes frenesy requirement (Π alone insufficient)	Theory	★★★★

No single counterexample is unambiguously fatal, but collectively they require AFET to: (1) incorporate frenesy D alongside Π, (2) treat the Π ↔ criticality coupling as non-monotonic, (3) make the coupling structure-dependent rather than universal, (4) exclude or redefine scope for QPTs and topological transitions, and (5) allow Θ_eff to be non-monotonic in Π.

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10. Multi-scale GENERIC provides the hierarchical structure AFET needs, with caveats

The Pavelka-Klika-Grmela framework (book: *Multiscale Thermo-Dynamics*, de Gruyter, 2018; Scinito Grmela, arXiv:2302.13164, 2023) organizes non-equilibrium theories as an oriented graph of autonomous mesoscopic theories connected by reduction links. arXiv arxiv Each vertex carries its own GENERIC structure: ẋ = L·E_x + Ξ_{x*}(x, X(x*)), with Poisson operator L (reversible) and dissipation potential Ξ (irreversible). cuni ResearchGate

The key mechanism for AFET is "rate thermodynamics." The dissipation potential Ξ at the upper level becomes the **rate entropy** S_rate for the lifting to the rate level: S_rate(x, X) = Ξ(x, X). This rate entropy drives the approach of thermodynamic forces toward the reduced description. The rate thermodynamic potential Ψ(X, J) = −Ξ(x, X) + ⟨X, J⟩ plays the role of a Lyapunov function, and its extremization (Ψ_X = 0) provides closure — expressing forces in terms of fluxes. arxiv For unforced systems, this is equivalent to the Onsager variational principle. arxiv

This mechanism has been demonstrated explicitly for: Boltzmann kinetic theory → fluid mechanics, arXiv the Peierls → Cattaneo → Fourier heat transfer hierarchy (PRE 83, 061134, 2011), APS Journals and the Grad hierarchy → extended hydrodynamics (PRE 95, 033121, 2017). The lack-of-fit reduction theorem (Pavelka, Klika, Grmela, J. Stat. Phys. 181, 19, 2020) proves GENERIC structure is preserved across levels. Springer Springer

Critical limitation: Explicit $\Pi_n \rightarrow \text{parameters}_{n+1} \rightarrow \Pi_{n+1}$ has been demonstrated only for **adjacent pairs of levels**, not for automated long chains. Standard GENERIC is Markovian; memory effects require adding state variables rather than being naturally accommodated (unlike Mori-Zwanzig). The Ehrenfest regularization (Physica D 399, 193, 2019) provides a pathway from reversible to irreversible dynamics through coarse-graining over a timescale τ, cuni arXiv but quantitative coarse-graining coefficients require case-by-case calculation.

The geometric aspects developed by Esen, Grmela, and Pavelka (arXiv:2205.10315, 2022) — cotangent lifts, contact geometry, holonomic decomposition — provide the deepest mathematical formalization. Contact geometry formulation embeds the second law directly into evolution equations, arXiv which is formally compatible with AFET's structure.

Assessment for AFET: Multi-scale GENERIC provides a rigorous formal backbone where entropy production at one level enters the next level's dynamics through the rate thermodynamics mechanism. However, it does not automatically generate the specific sigmoid-logistic coupling structure $A(t) = \sigma(\beta_{eff}(R - \Theta_{eff}))$ that AFET requires. AFET would need to demonstrate that GENERIC rate reduction, applied to an appropriate choice of state variables near a phase transition, produces this activation structure.

Synthesis and what the landscape means for AFET

The ten-problem survey reveals a theoretical landscape where entropy production is deeply connected to non-equilibrium critical behavior but cannot serve as a universal, monotonic modulator of criticality. The most important structural findings are:

EPR scaling at criticality is a genuine universal property (Caballero-Cates 2020; Paoluzzi 2022) with exponent $\theta_\sigma = \nu(4-d)$, DOI +2 but this universality describes Π as a diagnostic observable, not as a control parameter. DOI Resolver PubMed At the DP critical point, EPR is finite with a β-exponent cusp (Tomé-de Oliveira 2024). arXiv +3 These results support AFET's identification of EPR as carrying critical information but constrain how it enters the coupling operator.

The Aron-Chamon non-analytic potential provides the most scipost **promising formal pathway** to an entropy-modulated effective potential. The coefficient c_α vanishes at equilibrium and is controlled by non-equilibrium drive parameters correlated with Π. Deriving $c_\alpha(\Pi)$ explicitly would be a major advance for AFET.

Frenesy is indispensable. The Landauer blowtorch theorem and Maes' body of work establish theorem-level results that Π alone cannot characterize NESS behavior. ScienceDirect AFET's coupling operator F already includes D_d, which is well-motivated but experimentally challenging — currently measurable only in colloidal and mesoscopic electronic systems. ScienceDirect

The Fisher information–sigmoid connection is structural but formally incomplete. The fact that $\sigma'(\theta) = \sigma(\theta)(1-\sigma(\theta))$ is precisely the Fisher information of a Bernoulli model is mathematically exact and suggests deep structure, but the full chain from Wasserstein gradient flow to sigmoid activation for coarse-grained phase transition dynamics remains unproven.

Multi-scale GENERIC provides the best available formal hierarchical structure, with the rate thermodynamics mechanism channeling entropy production across levels. However, it requires augmentation with the specific activation structure and frenesy coupling that AFET proposes.

The principal falsification boundary at $\Pi = 0$ remains sharp: equilibrium criticality at $\Pi = 0$ is a well-defined boundary, but the counterexample analysis shows the boundary is richer than anticipated. Quantum phase transitions, topological transitions, and MBL transitions all occur at $\Pi = 0$ (or with Π destructive), requiring AFET to either restrict its scope or generalize Π to include quantum fluctuation contributions. The non-monotonic relationship between Π and criticality (quantum Zeno, Aguilera et al.) means the sigmoid modulation $A(t) = \sigma(\beta\_eff(R - \Theta\_eff))$ must allow for non-monotonic $\beta\_eff(\Pi)$ and $\Theta\_eff(\Pi)$, which the coupling operator F can in principle accommodate but which constrains the class of allowed functional forms.